

Transport properties of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory at finite coupling

Paolo Benincasa¹ and Alex Buchel^{1,2}

¹*Department of Applied Mathematics
University of Western Ontario
London, Ontario N6A 5B7, Canada*

²*Perimeter Institute for Theoretical Physics
Waterloo, Ontario N2J 2W9, Canada*

Abstract

Gauge theory-string theory duality describes strongly coupled $\mathcal{N} = 4$ supersymmetric $SU(n_c)$ Yang-Mills theory at finite temperature in terms of near extremal black 3-brane geometry in type IIB string theory. We use this correspondence to compute the leading correction in inverse 't Hooft coupling to the shear diffusion constant, bulk viscosity and the speed of sound in the large- n_c $\mathcal{N} = 4$ supersymmetric Yang-Mills theory plasma. The transport coefficients are extracted from the dispersion relation for the shear and the sound wave lowest quasinormal modes in the leading order α' -corrected black D3 brane geometry. We find the shear viscosity extracted from the shear diffusion constant to agree with result of [hep-th/0406264]; also, the leading correction to bulk viscosity and the speed of sound vanishes. Our computation provides a highly nontrivial consistency check on the hydrodynamic description of the α' -corrected nonextremal black branes in string theory.

October 2005

1 Introduction

The correspondence between gauge theories and string theory of Maldacena [1, 2] has become a valuable tool in analyzing near-equilibrium dynamics of strongly coupled gauge theory plasma [3–19]. The best studied example of strongly coupled thermal gauge theory plasma is that of the $\mathcal{N} = 4$ $SU(n_c)$ supersymmetric Yang-Mills theory (SYM). In the large- n_c limit, and at large 't Hooft coupling $g_{YM}^2 n_c \gg 1$, the holographic dual description of the $\mathcal{N} = 4$ plasma is in terms of near-extremal black 3-brane geometry in type IIB supergravity [20]. In this case one finds [5, 6, 8] the speed of sound c_s , the shear viscosity η , and the bulk viscosity ζ correspondingly

$$c_s = \frac{1}{\sqrt{3}}, \quad \eta = \frac{\pi}{8} n_c^2 T^3, \quad \zeta = 0. \quad (1.1)$$

In a hydrodynamic approximation to near-equilibrium dynamics of hot gauge theory plasma there are several distinct ways to extract the transport coefficients (1.1). First [5], the shear viscosity can be computed from the two-point correlation function of the stress-energy¹ tensor at zero spatial momentum via the Kubo formula

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{2\omega} \int dt d\bar{x} e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle. \quad (1.2)$$

Second [6], the diffusive channel two-point retarded correlation function of the stress energy tensor, for example,

$$G_{tx,tx}(\omega, q) = -i \int dt d\bar{x} e^{i\omega t - iqz} \theta(t) \langle [T_{tx}(x), T_{tx}(0)] \rangle \propto \frac{1}{i\omega - \mathcal{D}q^2} \quad (1.3)$$

has a pole at

$$\omega = -i\mathcal{D}q^2, \quad (1.4)$$

where the shear diffusion constant \mathcal{D} is

$$\mathcal{D} = \frac{\eta}{sT}, \quad (1.5)$$

with s being the entropy density of the gauge theory plasma. From the thermal field theory perspective it is clear that computation of the shear viscosity via Kubo relation (1.2), or from the pole of the stress-energy correlation function (1.3) (additionally using the equation of state to relate (1.5)) *must* give the same result. It is much less

¹Computation of the thermal correlation functions in the dual supergravity description was explained in [3, 4].

obvious that such an agreement should persist automatically also on the supergravity side. Thus, we regard above consistency of the hydrodynamic description of the black 3-branes in type IIB supergravity as a highly nontrivial check of the Maldacena correspondence [1] applied to near-equilibrium thermal gauge theories.

The situation with the sound wave propagation in the hydrodynamic limit is similar [8] (though perhaps less dramatic compare with shear viscosity given the conformal invariance of the $\mathcal{N} = 4$ SYM). The speed of sound can be computed from the equation of state as

$$c_s^2 = \frac{\partial P}{\partial \epsilon}, \quad (1.6)$$

where P and ϵ are correspondingly the pressure and the energy density of the strongly coupled gauge theory plasma which can be extracted from the thermodynamic properties of the black 3-branes [20]. Alternatively, all the transport coefficients (1.1) can be read off from the dispersion relation for the pole in the sound wave channel two-point retarded correlation function of the stress energy tensor, for example,

$$G_{tt,tt}(\omega, q) = -i \int dt d\bar{x} e^{i\omega t - iqz} \theta(t) \langle [T_{tt}(x), T_{tt}(0)] \rangle, \quad (1.7)$$

as

$$\omega(q) = c_s q - i \frac{2q^2}{3T} \frac{\eta}{s} \left(1 + \frac{3\zeta}{4\eta} \right). \quad (1.8)$$

Again, all these computations point to a consistent picture of a hydrodynamic description of the supergravity black 3-branes².

In this paper we prove that consistent hydrodynamic description of black 3-branes persists even once one include leading α' correction to type IIB supergravity from string theory [22–25], which translates into finite 't Hooft coupling correction on the $\mathcal{N} = 4$ SYM side of the Maldacena duality. To appreciate the nontrivial fact of the agreement we point to some features of α' -corrected description of the black 3-branes:

- including leading order α' correction, the entropy density of the black 3-branes differs from the Bekenstein-Hawking formula which relates the latter to the area of the horizon [26];
- the Hawking temperature of the black 3-branes as well as their equilibrium thermodynamic quantities, *i.e.*, the entropy, energy and the free energy, receives α' corrections [26, 27];

²Consistency of hydrodynamic description of more complicated examples of gauge theory-supergravity correspondence follows from [13, 17, 18, 21, 19].

- unlike the supergravity approximation [20], the radius of the S^5 of the α' corrected black 3-brane geometry is not constant [27].

We find that only properly accounting for *all* of the above facts one finds a consistent picture of the α' corrected black 3-brane hydrodynamics. Lastly, we strongly suspect that consistency of the hydrodynamics is sensitive to the structure of the α' corrections in type IIB string theory. Thus our computations can be helpful in determining exact structure of such corrections³.

The paper is organized as follows. In the next section we discuss our computational approach and present the results. In section 3 we apply the general computational scheme to the evaluation of the dispersion relation of the shear quasinormal mode in black 3-brane geometry without α' corrections. This was previously discussed in [16], though our approach highlights the use of the effective action rather than equations of motion⁴. In section 4 we discuss the main computational steps leading to the shear and the sound wave lowest quasinormal modes dispersion relations.

2 General computational approach and the results

In the context of gauge theory-string theory correspondence [1] poles of the finite temperature two-point retarded correlation functions of the stress-energy tensor can be identified with the quasinormal frequencies of the gravitational perturbations in the background string theory geometry [16]. Strictly speaking, such an identification has been made in the supergravity approximation to gauge theory-string theory correspondence, but as it is derived from the standard prescription for the computation of the correlation functions [28, 29, 3], we expect it to be valid beyond the supergravity approximation. In this paper we extract α' -corrected transport coefficients (1.1) from the α' -corrected dispersion relation for the lowest shear quasinormal mode (1.4) and the lowest sound wave quasinormal mode (1.8) in the α' -corrected black 3-brane geometry [26, 27].

We start with the tree level type IIB low-energy effective action in ten dimensions taking into account the leading order string corrections [22–25]

$$I = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4 \cdot 5!} (F_5)^2 + \dots + \gamma e^{-\frac{3}{2}\phi} W + \dots \right], \quad (2.1)$$

³We hope to report on this elsewhere.

⁴Using equations of motion is technically much more complicated in the presence of α' corrections.

$$\gamma = \frac{1}{8}\zeta(3)(\alpha')^3 ,$$

where

$$W = C^{hmnk}C_{pmnq}C_h^{rsp}C_{rsk}^q + \frac{1}{2}C^{hkmn}C_{pqmn}C_h^{rsp}C_{rsk}^q . \quad (2.2)$$

As in [26, 14] we assume that in a chosen scheme self-dual F_5 form does not receive order $(\alpha')^3$ corrections. In (2.1) ellipses stand for other fields not essential for the present analysis.

We represent ten dimensional background geometry describing γ -corrected black 3-branes by the following ansatz

$$\begin{aligned} ds_{10}^2 &= g_{\mu\nu}^{(0)} dx^\mu dx^\nu + c_4^2 (dS^5)^2 \\ &\equiv -c_1^2 dt^2 + c_2^2 (dx^2 + dy^2 + dz^2) + c_3^2 dr^2 + c_4^2 (dS^5)^2 , \end{aligned} \quad (2.3)$$

where $c_i = c_i(r)$ and $(dS^5)^2$ is a metric on a round five-sphere of unit radius. For the dilaton we assume $\phi = \phi(r)$ and for the five-form

$$F_5 = \mathcal{F}_5 + \star \mathcal{F}_5 , \quad \mathcal{F}_5 = -4 dvol_{S^5} . \quad (2.4)$$

In (2.4) the 5-form flux is chosen in such a way that $\gamma = 0$ solution corresponds to $c_4 = 1$. To leading order in γ , solution can be written explicitly [26, 27]

$$\begin{aligned} c_1 &= r \left(1 - \frac{r_0^4}{r^4}\right)^{1/2} e^{-\frac{5}{3}\nu} (1 + a + 4b) , \\ c_2 &= r e^{-\frac{5}{3}\nu} , \\ c_3 &= \frac{1}{r \left(1 - \frac{r_0^4}{r^4}\right)^{1/2}} e^{-\frac{5}{3}\nu} (1 + b) , \\ c_4 &= e^\nu , \end{aligned} \quad (2.5)$$

where to order $\mathcal{O}(\gamma^2)$

$$\begin{aligned} a &= -\gamma \frac{15r_0^4}{2r^4} \left(25\frac{r_0^4}{r^4} - 79\frac{r_0^8}{r^8} + 25\right) , \\ b &= \gamma \frac{15r_0^4}{2r^4} \left(5\frac{r_0^4}{r^4} - 19\frac{r_0^8}{r^8} + 5\right) , \\ \nu &= \gamma \frac{15r_0^8}{32r^8} \left(1 + \frac{r_0^4}{r^4}\right) . \end{aligned} \quad (2.6)$$

The dilaton ϕ also receives γ corrections, $\phi \propto \gamma$ [26]. It is easy to see that to order $\mathcal{O}(\gamma)$ gravitational perturbations do not mix with the dilaton perturbation; moreover

to study gravitational perturbations we can consistency set $\phi = 0$. The Hawking temperature corresponding to the metric (2.3) is [26]

$$T = T_0 (1 + 15\gamma) \equiv \frac{r_0}{\pi} (1 + 15\gamma) . \quad (2.7)$$

Next, consider perturbation of the five dimensional metric $g_{\mu\nu}^{(0)}$ (2.3)

$$g_{\mu\nu}^{(0)} \rightarrow g_{\mu\nu}^{(0)} + h_{\mu\nu} , \quad (2.8)$$

where it will be sufficient to assume that

$$h_{\mu\nu} = h_{\mu\nu}(t, z, r) = e^{-i\omega t + iqz} \hat{h}_{\mu\nu}(r) . \quad (2.9)$$

With the metric perturbation ansatz (2.9) we have $O(2)$ rotational symmetry in the xy plane. The latter symmetry guarantees that at the linearized level the following sets of fluctuations decouple from each other [6, 16]

$$\{h_{xy}\}, \quad \{h_{xx} - h_{yy}\} , \quad (2.10)$$

$$\{h_{tx}, h_{xz}, h_{xr}\} , \quad \{h_{ty}, h_{yz}, h_{yr}\} , \quad (2.11)$$

$$\{h_{tt}, h_{tz}, h_{tr}, h_{aa} \equiv h_{xx} + h_{yy}, h_{zz}, h_{zr}, h_{rr}\} . \quad (2.12)$$

Scalar channel fluctuations (2.10) were studied in [14] leading (using the Kubo relation (1.2)) to the following prediction for the shear viscosity to the entropy density ratio

$$\frac{\eta}{s} = \frac{1}{4\pi} (1 + 135\gamma) . \quad (2.13)$$

In this paper we study shear channel (2.11), and the sound channel (2.12) fluctuations. Effective action for the fluctuations (2.11) and (2.12) can be obtained by expanding the supergravity action (2.1) around the background⁵ (2.3) to quadratic order in $h_{\mu\nu}$. Though we can always choose the gauge

$$h_{tr} = h_{xr} = h_{yr} = h_{zr} = h_{rr} = 0 , \quad (2.14)$$

⁵There is a subtlety in evaluating the action with a self-dual 5-form background. The correct way to do this is to assume that F_5 has components only along S^5 and double that contribution in the 10d effective action [30].

doing so on the level of the effective action for the fluctuations would lead to missing important constraints, *i.e.*, equations of motion coming from the variation of the action with respect to $\{h_{tr}, h_{xr}, h_{yr}, h_{zr}, h_{rr}\}$. As we explicitly demonstrate on a simple example in the next section these constraint equations are crucial in decoupling gauge invariant fluctuations. Rather, the correct way is to impose the gauge fixing (2.14) on the level of equations of motion for the fluctuations.

Without loss of generality, for the shear channel we consider metric perturbations $\{h_{tx}, h_{xz}, h_{xr}\}$. Imposing the gauge condition $h_{xr}=0$ on the equations of motion and further introducing

$$\hat{h}_{tx}(r) = r^2 H_{tx}(r), \quad \hat{h}_{xz}(r) = r^2 H_{xz}(r), \quad (2.15)$$

we find that the shear channel gauge invariant combination [16]

$$Z_{shear} = qH_{tx} + \omega H_{xz} \quad (2.16)$$

decouples to order $\mathcal{O}(\gamma)$. The spectrum of quasinormal modes is determined [16] by imposing the incoming wave boundary condition at the horizon $r \rightarrow r_0+$, and the Dirichlet condition at the boundary $r \rightarrow +\infty$ in the γ -deformed black 3-brane geometry (2.3) on Z_{shear} . The main steps of the computation are discussed in section 4.1. For the lowest shear quasinormal mode (in the hydrodynamic approximation) we find to order $\mathcal{O}(\gamma)$

$$\mathfrak{w} = -i \Gamma_\eta \mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^3), \quad (2.17)$$

where

$$\Gamma_\eta = \frac{1}{2} + 60\gamma + \mathcal{O}(\gamma^2), \quad (2.18)$$

and we additionally introduced⁶

$$\mathfrak{w} = \frac{\omega}{2\pi T_0}, \quad \mathfrak{q} = \frac{q}{2\pi T_0}. \quad (2.19)$$

From (1.5), (2.7), (2.18), (2.19) we find

$$\frac{\eta}{s} = T\mathcal{D} = T \times \frac{1}{2\pi T_0} \times \Gamma_\eta = \frac{1}{4\pi} (1 + 135\gamma + \mathcal{O}(\gamma^2)), \quad (2.20)$$

in precise agreement with (2.13) reported in [14].

⁶As will be clear from the discussion in section 4, \mathfrak{w} and \mathfrak{q} are the natural dimensionless parameters describing quasinormal modes.

There is additional subtlety in computing the lowest quasinormal frequency in the sound channel. Similar to the shear channel⁷ we first derive from the effective action for the fluctuations equations of motion, and after that impose the gauge condition

$$h_{tr} = h_{zr} = h_{rr} = 0. \quad (2.21)$$

As explained in [17], for a general five-dimensional Einstein frame background geometry with the metric

$$d\hat{s}_5^2 = -\hat{c}_1^2 dt^2 + \hat{c}_2^2 (dx^2 + dy^2 + dz^2) + \hat{c}_3^2 dr^2, \quad (2.22)$$

with $\hat{c}_i = \hat{c}_i(r)$, in the gauge (2.21), and reparameterizing metric perturbations (2.9) as

$$\hat{h}_{tt} = \hat{c}_1^2 H_{tt}, \quad \hat{h}_{tz} = \hat{c}_2^2 H_{tz}, \quad \hat{h}_{aa} = \hat{c}_2^2 H_{aa}, \quad \hat{h}_{zz} = \hat{c}_2^2 H_{zz}, \quad (2.23)$$

the gauge invariant gravitational perturbation is given by

$$Z_{sound} = 4 \frac{q}{\omega} H_{tz} + 2 H_{zz} - H_{aa} \left(1 - \frac{q^2}{\omega^2} \frac{\hat{c}'_1 \hat{c}_1}{\hat{c}'_2 \hat{c}_2} \right) + 2 \frac{q^2}{\omega^2} \frac{\hat{c}_1^2}{\hat{c}_2^2} H_{tt}, \quad (2.24)$$

and thus (in the absence of matter sector) must have decoupled equation of motion. In the absence of γ -corrections, the ten-dimensional Einstein frame reduces to the five-dimensional Einstein frame, so in defining Z_{sound} we can simply take $\hat{c}_i = c_i$. This is no longer the case with $\gamma \neq 0$, as in this case the S^5 warp factor $\propto c_4^2$ is no longer constant. Indeed, we find that defining Z_{sound} as in (2.23) produces decoupled equation of motion only after \hat{c}_i are rescaled as appropriate for the five-dimensional Einstein frame, namely

$$\hat{c}_i = c_4^{5/3} c_i. \quad (2.25)$$

Again, the spectrum of quasinormal modes is determined by imposing the incoming wave boundary condition at the horizon, and the Dirichlet condition at the boundary in the γ -deformed background geometry (2.3) on Z_{sound} . For the lowest shear quasinormal mode (in the hydrodynamic approximation) we find to order $\mathcal{O}(\gamma)$

$$\mathfrak{w} = c_s \mathfrak{q} - i \Gamma_{sound} \mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^3), \quad (2.26)$$

where

$$\begin{aligned} c_s &= \frac{1}{\sqrt{3}} + \mathcal{O}(\gamma^2), \\ \Gamma_\eta &= \frac{1}{3} + 40\gamma + \mathcal{O}(\gamma^2). \end{aligned} \quad (2.27)$$

⁷The main computational steps are presented in section 4.2.

Given (1.8), (2.7), (2.19), (2.20) we conclude from (2.27) that the bulk viscosity of the strongly coupled $\mathcal{N} = 4$ plasma is

$$\zeta = \mathcal{O}(\gamma^2). \quad (2.28)$$

Of course, as finite γ -corrections translate into 't Hooft coupling corrections on the gauge theory side of the Maldacena correspondence, from the field theory perspective (given that the latter is conformal) we immediately conclude that $c_s^2 = \frac{1}{3}$ and $\zeta = 0$ independent of the 't Hooft coupling. We showed here that the dual string theory description reproduces this fact as well, albeit in a highly nontrivial fashion which is moreover consistent with shear viscosity computation (2.20) and alternative analysis in [14].

3 Diffusion constant of the black 3-branes hydrodynamics: the effective action approach

Consider the shear channel gravitational perturbations $\{h_{tx}, h_{xz}, h_{xr}\}$ in the absence of α' corrections, *i.e.*, setting $\gamma = 0$. This was previously discussed in [6], where equations of motion for the fluctuations $\{h_{tx}, h_{xz}\}$ in the gauge $h_{xr} = 0$ were derived as perturbation of the full type IIB supergravity equations of motion. Such an approach becomes technically very difficult in the presence of γ corrections: one needs to derive equations of motion for the deformed effective type IIB supergravity action (2.1). In the latter case we find it much easier to derive first the effective action describing the fluctuations, and then derive the equations of motion from this action. The effective action for the fluctuations can be obtained by simply evaluating (2.1) to quadratic order in metric perturbations (2.8). The important point we want to stress here is that the gauge fixing condition $h_{xr} = 0$ can not be imposed on the level of action. If we do this, we obtain two second order ODE's (coming from variation of the action with respect to $\{h_{tx}, h_{xz}\}$)

$$\begin{aligned} 0 &= H''_{tx} - \frac{1}{x} H'_{tx} - \frac{\mathfrak{q}}{(1-x^2)^{3/2}} \left(\mathfrak{w} H_{xz} + \mathfrak{q} H_{tx} \right), \\ 0 &= H''_{xz} + \frac{1}{x} H'_{xz} + \frac{\mathfrak{w}}{x^2(1-x^2)^{3/2}} \left(\mathfrak{w} H_{xz} + \mathfrak{q} H_{tx} \right), \end{aligned} \quad (3.1)$$

where $H_{...} = H_{...}(x)$, and all the derivatives are with respect to

$$x \equiv \left(1 - \frac{r_0^4}{r^4} \right)^{1/2}. \quad (3.2)$$

It is easy to see that given (3.2) equation of motion for

$$Z_{shear}(x) = \mathfrak{q}H_{tx}(x) + \mathfrak{w}H_{xz}(x) \quad (3.3)$$

does not decouple. On the other hand, if we impose the gauge fixing condition $h_{xr} = 0$ on the level of equations of motion, we obtain an extra constraint equation coming from the variation of the effective action for the fluctuations with respect to h_{xr}

$$0 = \mathfrak{w}H'_{tx} + \mathfrak{q}x^2H'_{xz}. \quad (3.4)$$

Notice that (3.4) is consistent with (3.1). Given (3.4) we can now obtain the decoupled equation of motion for Z_{shear}

$$0 = Z''_{shear} + \frac{x^2\mathfrak{q}^2 + \mathfrak{w}^2}{x(\mathfrak{w}^2 - x^2\mathfrak{q}^2)} Z'_{shear} + \frac{\mathfrak{w}^2 - x^2\mathfrak{q}^2}{x^2(1 - x^2)^{3/2}} Z_{shear}. \quad (3.5)$$

The incoming boundary condition at the horizon ($x \rightarrow 0_+$) implies that

$$Z_{shear}(x) = x^{-i\mathfrak{w}} z_{shear}(x), \quad (3.6)$$

where $z_{shear}(x)$ is regular at the horizon. Without loss of generality we can assume

$$z_{shear} \Big|_{x \rightarrow 0_+} = 1, \quad (3.7)$$

the spectrum of quasinormal frequencies is then determined by imposing a Dirichlet condition at the boundary [16]

$$z_{shear} \Big|_{x \rightarrow 1_-} = 0. \quad (3.8)$$

In the hydrodynamic approximation ($\mathfrak{w} \ll 1$ and $\mathfrak{q} \ll 1$) the solution can be written in the ansatz

$$z_{shear} = z_{shear}^{(0)} + i \mathfrak{q} z_{shear}^{(1)} + \mathcal{O}(\mathfrak{q}^2), \quad (3.9)$$

where $z_{shear}^{(0)}$, $z_{shear}^{(1)}$ are invariant under the scaling $\mathfrak{w} \rightarrow \lambda \mathfrak{w}$, $\mathfrak{q} \rightarrow \lambda \mathfrak{q}$ with constant λ . Substituting (3.9) into (3.5), and we find [6, 16]

$$z_{shear}^{(0)} = 1, \quad z_{shear}^{(1)} = \frac{1}{2} \frac{\mathfrak{q}}{\mathfrak{w}} x^2, \quad (3.10)$$

which from (3.8) determines the lowest shear quasinormal frequency as [16]

$$\mathfrak{w} = -i \frac{1}{2} \mathfrak{q}^2. \quad (3.11)$$

4 Transport properties of black 3-branes at $\mathcal{O}((\alpha')^3)$ order

The general computational scheme for deriving equations of motion for the metric perturbation and decoupling the gauge invariant combinations for these perturbations is explained in section 2. The analysis is straightforward, though quite tedious. As in the previous section, computations are simplified using the radial coordinate x , defined by (3.2). Both for the shear mode (2.16) and the sound mode (2.24) gauge invariant combination of metric perturbations we find that the corresponding equations of motion decouple. These equations can be expanded perturbatively in γ , provided we introduce

$$\begin{aligned} Z_{\text{shear}} &= Z_{\text{shear},0} + \gamma Z_{\text{shear},1} + \mathcal{O}(\gamma^2), \\ Z_{\text{sound}} &= Z_{\text{sound},0} + \gamma Z_{\text{sound},1} + \mathcal{O}(\gamma^2). \end{aligned} \quad (4.1)$$

The incoming wave boundary conditions are set up at the level of the leading order in γ , thus it is not a surprise that a natural dimensionless frequency \mathfrak{w} and a momentum \mathfrak{q} are introduced (see eq. (2.19)) with respect to T_0 , rather than the α' -corrected Hawking temperature T of the black branes (2.7).

4.1 Shear quasinormal mode

For the shear channel fluctuations we find

$$\begin{aligned} 0 &= Z''_{\text{shear},0} + \frac{x^2 \mathfrak{q}^2 + \mathfrak{w}^2}{x(\mathfrak{w}^2 - x^2 \mathfrak{q}^2)} Z'_{\text{shear},0} + \frac{\mathfrak{w}^2 - x^2 \mathfrak{q}^2}{x^2(1 - x^2)^{3/2}} Z_{\text{shear},0}, \\ 0 &= Z''_{\text{shear},1} + \frac{x^2 \mathfrak{q}^2 + \mathfrak{w}^2}{x(\mathfrak{w}^2 - x^2 \mathfrak{q}^2)} Z'_{\text{shear},1} + \frac{\mathfrak{w}^2 - x^2 \mathfrak{q}^2}{x^2(1 - x^2)^{3/2}} Z_{\text{shear},1} + J_{\text{shear},0}, \end{aligned} \quad (4.2)$$

where the source $J_{\text{shear},0}$ is a functional of the zero's order shear mode $Z_{\text{shear},0}$

$$\begin{aligned} J_{\text{shear},0} &= \mathcal{C}_{\text{shear}}^{(4)} \frac{d^4 Z_{\text{shear},0}}{dx^4} + \mathcal{C}_{\text{shear}}^{(3)} \frac{d^3 Z_{\text{shear},0}}{dx^3} + \mathcal{C}_{\text{shear}}^{(2)} \frac{d^2 Z_{\text{shear},0}}{dx^2} + \mathcal{C}_{\text{shear}}^{(1)} \frac{d Z_{\text{shear},0}}{dx} \\ &\quad + \mathcal{C}_{\text{shear}}^{(0)} Z_{\text{shear},0}. \end{aligned} \quad (4.3)$$

The coefficients $\mathcal{C}_{\text{shear}}^{(i)}$ are given explicitly in appendix A. In the hydrodynamic approximation we look for the solution for $Z_{\text{shear},i}$ in the following ansatz

$$\begin{aligned} Z_{\text{shear},0} &= x^{-i\mathfrak{w}} \left(z_{\text{shear},0}^{(0)} + i\mathfrak{q} z_{\text{shear},0}^{(1)} + \mathcal{O}(\mathfrak{q}^2) \right), \\ Z_{\text{shear},1} &= x^{-i\mathfrak{w}} \left(z_{\text{shear},1}^{(0)} + i\mathfrak{q} z_{\text{shear},1}^{(1)} + \mathcal{O}(\mathfrak{q}^2) \right), \end{aligned} \quad (4.4)$$

where $z_{shear,i}^{(j)}$ are regular at the horizon, and satisfy the following boundary conditions

$$z_{shear,0}^{(0)} \Big|_{x \rightarrow 0_+} = 1, \quad z_{shear,0}^{(1)} \Big|_{x \rightarrow 0_+} = z_{shear,1}^{(0)} \Big|_{x \rightarrow 0_+} = z_{shear,1}^{(1)} \Big|_{x \rightarrow 0_+} = 0. \quad (4.5)$$

Explicit solution of (4.2) subject to boundary conditions (4.5) takes form

$$z_{shear,0}^{(0)} = 1, \quad z_{shear,0}^{(1)} = \frac{1}{2} \frac{\mathfrak{q}}{\mathfrak{w}} x^2, \quad (4.6)$$

$$\begin{aligned} z_{shear,1}^{(0)} &= \frac{25}{16} x^2 (x^4 - 4x^2 + 5), \\ z_{shear,1}^{(1)} &= -\frac{1}{32\mathfrak{q}\mathfrak{w}} x^2 \left(\mathfrak{q}^2 (-240 - 1565x^2 - 860x^4 + 695x^6) \right. \\ &\quad \left. + 16\mathfrak{w}^2 (594 - 264x^2 + 43x^4) \right). \end{aligned} \quad (4.7)$$

Imposing the Dirichlet condition on $x^{i\mathfrak{w}} Z_{shear,0}$ at the boundary determines the lowest shear quasinormal frequency (2.17).

4.2 Sound wave quasinormal mode

For the sound channel fluctuations we find

$$\begin{aligned} 0 &= Z''_{sound,0} - \frac{(3x^2 - 2)\mathfrak{q}^2 + 3\mathfrak{w}^2}{x(-3\mathfrak{w}^2 + (x^2 + 2)\mathfrak{q}^2)} Z'_{sound,0} \\ &\quad - \frac{\mathfrak{q}^4 x^2 (x^2 + 2) - 2\mathfrak{q}^2 \mathfrak{w}^2 (2x^2 + 2) - 4x^2 (1 - x^2)^{3/2} \mathfrak{q}^2 + 3\mathfrak{w}^4}{x^2 (1 - x^2)^{3/2} ((x^2 + 2)\mathfrak{q}^2 - 3\mathfrak{w}^2)} Z_{sound,0}, \\ 0 &= Z''_{sound,1} - \frac{(3x^2 - 2)\mathfrak{q}^2 + 3\mathfrak{w}^2}{x(-3\mathfrak{w}^2 + (x^2 + 2)\mathfrak{q}^2)} Z'_{sound,1} \\ &\quad - \frac{\mathfrak{q}^4 x^2 (x^2 + 2) - 2\mathfrak{q}^2 \mathfrak{w}^2 (2x^2 + 2) - 4x^2 (1 - x^2)^{3/2} \mathfrak{q}^2 + 3\mathfrak{w}^4}{x^2 (1 - x^2)^{3/2} ((x^2 + 2)\mathfrak{q}^2 - 3\mathfrak{w}^2)} Z_{sound,1} + J_{sound,0}, \end{aligned} \quad (4.8)$$

where the source $J_{sound,0}$ is a functional of the zero's order sound mode $Z_{sound,0}$

$$\begin{aligned} J_{sound,0} &= \mathcal{C}_{sound}^{(4)} \frac{d^4 Z_{sound,0}}{dx^4} + \mathcal{C}_{sound}^{(3)} \frac{d^3 Z_{sound,0}}{dx^3} + \mathcal{C}_{sound}^{(2)} \frac{d^2 Z_{sound,0}}{dx^2} + \mathcal{C}_{sound}^{(1)} \frac{d Z_{sound,0}}{dx} \\ &\quad + \mathcal{C}_{sound}^{(0)} Z_{sound,0}. \end{aligned} \quad (4.9)$$

The coefficients $\mathcal{C}_{sound}^{(i)}$ are given explicitly in appendix B. In the hydrodynamic approximation we look for the solution for $Z_{sound,i}$ in the following ansatz

$$\begin{aligned} Z_{sound,0} &= x^{-i\mathfrak{w}} \left(z_{sound,0}^{(0)} + i\mathfrak{q}z_{sound,0}^{(1)} + \mathcal{O}(\mathfrak{q}^2) \right), \\ Z_{sound,1} &= x^{-i\mathfrak{w}} \left(z_{sound,1}^{(0)} + i\mathfrak{q}z_{sound,1}^{(1)} + \mathcal{O}(\mathfrak{q}^2) \right), \end{aligned} \quad (4.10)$$

where $z_{sound,i}^{(j)}$ are regular at the horizon, and satisfy the following boundary conditions

$$z_{sound,0}^{(0)} \Big|_{x \rightarrow 0_+} = 1, \quad z_{sound,0}^{(1)} \Big|_{x \rightarrow 0_+} = z_{sound,1}^{(0)} \Big|_{x \rightarrow 0_+} = z_{sound,1}^{(1)} \Big|_{x \rightarrow 0_+} = 0. \quad (4.11)$$

Explicit solution of (4.8) subject to boundary conditions (4.11) takes form

$$z_{sound,0}^{(0)} = \frac{3\mathfrak{w}^2 + (x^2 - 2)\mathfrak{q}^2}{3\mathfrak{w}^2 - 2\mathfrak{q}^2}, \quad z_{sound,0}^{(1)} = \frac{2\mathfrak{w}\mathfrak{q}x^2}{3\mathfrak{w}^2 - 2\mathfrak{q}^2}, \quad (4.12)$$

$$\begin{aligned} z_{sound,1}^{(0)} &= \frac{5x^2}{16(3\mathfrak{w}^2 - 2\mathfrak{q}^2)^2} \left(\mathfrak{q}^4 (2404 + 446x^2 - 4164x^4 + 2006x^6) \right. \\ &\quad \left. - 3\mathfrak{w}^2\mathfrak{q}^2 (1588 + 183x^2 - 2072x^4 + 1003x^6) + 45\mathfrak{w}^4 (5 - 4x^2 + x^4) \right), \\ z_{sound,1}^{(1)} &= \frac{\mathfrak{w}x^2}{8\mathfrak{q}(3\mathfrak{w}^2 - 2\mathfrak{q}^2)^2} \left(\mathfrak{q}^4 (-13344 + 5846x^2 - 4520x^4 + 1734x^6) \right. \\ &\quad \left. - 3\mathfrak{w}^2\mathfrak{q}^2 (-9744 + 5035x^2 - 2604x^4 + 867x^6) \right. \\ &\quad \left. - 36\mathfrak{w}^4 (594 - 264x^2 + 43x^4) \right). \end{aligned} \quad (4.13)$$

Imposing the Dirichlet condition on $x^{i\mathfrak{w}}Z_{sound,0}$ at the boundary determines the lowest sound quasinormal frequency (2.26).

Appendix

A Coefficients of $J_{shear,0}$

$$\mathcal{C}_{shear}^{(4)} = 45(1 - x^2)^4 \quad (A.1)$$

$$\mathcal{C}_{shear}^{(3)} = 90(1 - x^2)^3 \frac{(\mathfrak{q}^2(7x^2 + 1) - 8\mathfrak{w}^2)x}{-\mathfrak{q}^2x^2 + \mathfrak{w}^2} \quad (A.2)$$

$$\begin{aligned} \mathcal{C}_{shear}^{(2)} = & -\frac{1}{8x^2(-\mathfrak{q}^2x^2 + \mathfrak{w}^2)} \left(5(1-x^2)^2(\mathfrak{q}^2x^2(2803x^4 + 1018x^2 + 216) - \mathfrak{w}^2(72 \right. \\ & \left. + 3811x^4 + 154x^2)) + 16(1-x^2)^{5/2}(\mathfrak{q}^2x^2 - \mathfrak{w}^2)(45\mathfrak{w}^2 + 83\mathfrak{q}^2x^2) \right) \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \mathcal{C}_{shear}^{(1)} = & -\frac{1}{8x^3(-\mathfrak{q}^2x^2 + \mathfrak{w}^2)^2} \left(-5\mathfrak{q}^4x^4(1-x^2)(1105x^6 - 2411x^4 - 802x^2 - 216) \right. \\ & - 4\mathfrak{w}^2\mathfrak{q}^2x^2(-8792x^6 + 1467x^8 + 8045x^4 + 60x^2 - 1080) - \mathfrak{w}^4(-13479x^4 + 3353x^8 \\ & + 2930x^2 + 5876x^6 + 2520) + 16(1-x^2)^{3/2}(83x^6(4x^2 + 1)\mathfrak{q}^6 - \mathfrak{w}^2\mathfrak{q}^4x^4(497x^2 - 372) \\ & \left. - \mathfrak{q}^2\mathfrak{w}^4x^2(162x^2 + 353) + 45\mathfrak{w}^6(3x^2 + 2)) \right) \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} \mathcal{C}_{shear}^{(0)} = & -\frac{1}{8x^4(-\mathfrak{q}^2x^2 + \mathfrak{w}^2)} \left(152\mathfrak{q}^6x^6(-1+x^2) - 1480\mathfrak{w}^2\mathfrak{q}^4x^4(-1+x^2) \right. \\ & + 4x^2\mathfrak{q}^2(-242\mathfrak{w}^4 + 242\mathfrak{w}^4x^2 - 75x^8 + 100x^6) + 10\mathfrak{w}^2(-36\mathfrak{w}^4 + 36\mathfrak{w}^4x^2 + 45x^8 \\ & - 80x^6 + 25x^4) + (-5\mathfrak{q}^4x^6(-831x^2 + 274 + 677x^4) + 2\mathfrak{w}^2\mathfrak{q}^2x^2(-1858x^2 + 4453x^6 \\ & \left. - 5295x^4 + 3600) - (4320 + 1250x^2 + 6097x^6 - 10467x^4)\mathfrak{w}^4)(1-x^2)^{-1/2} \right) \end{aligned} \quad (\text{A.5})$$

B Coefficients of $J_{sound,0}$

$$\mathcal{C}_{sound}^{(4)} = \frac{2\mathfrak{q}^2(183x^2 + 29) + 333\mathfrak{w}^2}{3(2\mathfrak{q}^2(x^2 - 1) + 3\mathfrak{w}^2)}(1-x^2)^4 \quad (\text{B.1})$$

$$\begin{aligned} \mathcal{C}_{sound}^{(3)} = & \frac{2(x^2 - 1)^3}{3x(\mathfrak{q}^2(x^2 + 2) - 3\mathfrak{w}^2)(2\mathfrak{q}^2(x^2 - 1) + 3\mathfrak{w}^2)^2} \left(4\mathfrak{q}^6(x^2 - 1)(872x^6 + 2486x^4 \right. \\ & - 567x^2 - 58) - 6\mathfrak{w}^2\mathfrak{q}^4(557x^6 - 7955x^4 + 2386x^2 - 106) - 9\mathfrak{w}^4\mathfrak{q}^2(4237x^4 - 3875x^2 \\ & \left. + 386) - 2997\mathfrak{w}^6(9x^2 - 1) \right) \end{aligned} \quad (\text{B.2})$$

$$\begin{aligned} \mathcal{C}_{sound}^{(2)} = & \frac{1}{(24x^2(\mathfrak{q}^2(x^2+2)-3\mathfrak{w}^2))(2\mathfrak{q}^2(x^2-1)+3\mathfrak{w}^2)^3} \left(8(17357x^{10} + 64126x^8 \right. \\ & - 125343x^6 + 33528x^4 + 4468x^2 + 464)(x^2-1)^3\mathfrak{q}^8 - 12\mathfrak{w}^2(12419x^{10} - 416676x^8 \\ & + 639279x^6 - 205814x^4 - 18792x^2 + 384)(x^2-1)^2\mathfrak{q}^6 - 18\mathfrak{w}^4(x^2-1)(183535x^{10} \\ & - 1069144x^8 + 1301579x^6 - 428618x^4 + 14112x^2 + 3936)\mathfrak{q}^4 - 27\mathfrak{w}^6(x^2-1)(359001x^8 \\ & - 732179x^6 + 440634x^4 - 71592x^2 - 4864)\mathfrak{q}^2 - 243\mathfrak{w}^8(x^2-1)(22327x^6 - 28997x^4 \\ & + 6974x^2 + 296) - 16(2\mathfrak{q}^2(x^2-1) + 3\mathfrak{w}^2)(\mathfrak{q}^2(x^2+2) - 3\mathfrak{w}^2)(1-x^2)^{5/2}(4x^2(x^2-1) \\ & \times (125x^2 - 87)\mathfrak{q}^6 + 4\mathfrak{w}^2(273x^4 - 86x^2 + 29)\mathfrak{q}^4 - 3\mathfrak{w}^4(159x^2 - 164)\mathfrak{q}^2 - 999\mathfrak{w}^6) \left. \right) \end{aligned} \quad (B.3)$$

$$\begin{aligned}
\mathcal{C}_{sound}^{(0)} = & \frac{1}{24x^4(2\mathfrak{q}^2(x^2-1) + 3\mathfrak{w}^2)^3(\mathfrak{q}^2(x^2+2) - 3\mathfrak{w}^2)^3} \left(-64x^4(643x^2 - 779)(x^2-1)^3 \right. \\
& \times (x^2+2)^3\mathfrak{q}^{16} + 32x^2\mathfrak{w}^2(5747x^6 - 31591x^4 + 25688x^2 + 696)(x^2-1)^2(x^2+2)^2\mathfrak{q}^{14} \\
& - 16(x^2-1)(343159x^{18} + 359756x^{16} - 2430770x^{14} - 19905\mathfrak{w}^4x^{12} + 1208858x^{12} \\
& - 216211\mathfrak{w}^4x^{10} + 2130051x^{10} - 1974730x^8 + 212468\mathfrak{w}^4x^8 + 789188\mathfrak{w}^4x^6 + 317076x^6 \\
& - 704624\mathfrak{w}^4x^4 + 46600x^4 - 43376\mathfrak{w}^4x^2 + 928\mathfrak{w}^4)\mathfrak{q}^{12} + 24\mathfrak{w}^2(x^2-1)(385565x^{16} \\
& - 4651541x^{14} + 3705393x^{12} + 6897979x^{10} - 73557\mathfrak{w}^4x^{10} - 7874224x^8 + 121796\mathfrak{w}^4x^8 \\
& + 1319004x^6 + 803308\mathfrak{w}^4x^6 - 855456\mathfrak{w}^4x^4 + 217824x^4 - 96080\mathfrak{w}^4x^2 + 1088\mathfrak{w}^4)\mathfrak{q}^{10} \\
& + 36\mathfrak{w}^4(x^2-1)(2216157x^{14} - 3463628x^{12} - 8055387x^{10} - 26682\mathfrak{w}^4x^8 + 12127416x^8 \\
& - 2103232x^6 - 408776\mathfrak{w}^4x^6 - 547176x^4 + 579864\mathfrak{w}^4x^4 + 117600\mathfrak{w}^4x^2 + 8480\mathfrak{w}^4)\mathfrak{q}^8 \\
& + 54\mathfrak{w}^6(26240\mathfrak{w}^4 + 10762468x^8 - 803584x^4 + 2980096x^{12} + 956891x^{14} - 462808x^6 \\
& - 13364213x^{10} + 122304\mathfrak{w}^4x^4 - 290744\mathfrak{w}^4x^6 + 58560\mathfrak{w}^4x^2 + 83640\mathfrak{w}^4x^8)\mathfrak{q}^6 \\
& - 162\mathfrak{w}^8(15440\mathfrak{w}^4 + 355529x^{12} + 1905537x^8 - 2260254x^{10} - 303468x^4 + 349906x^6 \\
& - 2056\mathfrak{w}^4x^4 - 15120\mathfrak{w}^4x^6 + 1736\mathfrak{w}^4x^2)\mathfrak{q}^4 - 486\mathfrak{w}^{10}(-4208\mathfrak{w}^4 + 150699x^{10} + 45576x^4 \\
& - 93108x^8 - 111717x^6 + 2684\mathfrak{w}^4x^2 + 1524\mathfrak{w}^4x^4)\mathfrak{q}^2 + 4374\mathfrak{w}^{12}(-148\mathfrak{w}^4 - 400x^6 \\
& + 148\mathfrak{w}^4x^2 + 125x^4 + 225x^8) - (\mathfrak{q}^2(x^2+2) - 3\mathfrak{w}^2)(1-x^2)^{-1/2}(8x^4(117035x^{10} \\
& + 280248x^8 - 600005x^6 - 185858x^4 + 519204x^2 - 114424)(x^2-1)^2\mathfrak{q}^{12} - 4\mathfrak{w}^2(x^2-1) \\
& \times (403063x^{14} - 5091874x^{12} + 4346503x^{10} + 5410988x^8 - 6467736x^6 + 1273520x^4 \\
& + 35760x^2 - 7424)\mathfrak{q}^{10} - 6\mathfrak{w}^4(10631928x^8 + 2466759x^{14} + 1592283x^{10} - 7034846x^{12} \\
& - 8874092x^6 + 974760x^4 + 195888x^2 - 1280)\mathfrak{q}^8 - 27\mathfrak{w}^6(-797920x^8 + 485056x^4 \\
& + 475731x^{12} + 927244x^6 - 918927x^{10} - 23040 - 115744x^2)\mathfrak{q}^6 - 27\mathfrak{w}^8(-269131x^8 \\
& + 473495x^{10} - 1348292x^4 + 908224x^6 + 70400 + 149104x^2)\mathfrak{q}^4 - 81\mathfrak{w}^{10}(314448x^4 \\
& + 56787x^8 - 321287x^6 - 26560 - 30588x^2)\mathfrak{q}^2 - 729\mathfrak{w}^{12}(1184 + 2665x^6 - 4027x^4 \\
& + 778x^2)) \right) \tag{B.5}
\end{aligned}$$

Acknowledgments

Research at Perimeter Institute is supported in part by funds from NSERC of Canada. AB gratefully acknowledges support by NSERC Discovery grant.

References

- [1] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” *Adv. Theor. Math. Phys.* **2**, 231 (1998) [*Int. J. Theor. Phys.* **38**, 1113 (1999)] [[arXiv:hep-th/9711200](https://arxiv.org/abs/hep-th/9711200)].
- [2] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, *Phys. Rept.* **323**, 183 (2000) [[arXiv:hep-th/9905111](https://arxiv.org/abs/hep-th/9905111)].
- [3] D. T. Son and A. O. Starinets, “Minkowski-space correlators in AdS/CFT correspondence: Recipe and applications,” *JHEP* **0209**, 042 (2002) [[arXiv:hep-th/0205051](https://arxiv.org/abs/hep-th/0205051)].
- [4] C. P. Herzog and D. T. Son, “Schwinger-Keldysh propagators from AdS/CFT correspondence,” *JHEP* **0303**, 046 (2003) [[arXiv:hep-th/0212072](https://arxiv.org/abs/hep-th/0212072)].
- [5] G. Policastro, D. T. Son and A. O. Starinets, “The shear viscosity of strongly coupled $N = 4$ supersymmetric Yang-Mills plasma,” *Phys. Rev. Lett.* **87**, 081601 (2001) [[arXiv:hep-th/0104066](https://arxiv.org/abs/hep-th/0104066)].
- [6] G. Policastro, D. T. Son and A. O. Starinets, “From AdS/CFT correspondence to hydrodynamics,” *JHEP* **0209**, 043 (2002) [[arXiv:hep-th/0205052](https://arxiv.org/abs/hep-th/0205052)].
- [7] C. P. Herzog, “The hydrodynamics of M-theory,” *JHEP* **0212**, 026 (2002) [[arXiv:hep-th/0210126](https://arxiv.org/abs/hep-th/0210126)].
- [8] G. Policastro, D. T. Son and A. O. Starinets, “From AdS/CFT correspondence to hydrodynamics. II: Sound waves,” *JHEP* **0212**, 054 (2002) [[arXiv:hep-th/0210220](https://arxiv.org/abs/hep-th/0210220)].
- [9] C. P. Herzog, “The sound of M-theory,” *Phys. Rev. D* **68**, 024013 (2003) [[arXiv:hep-th/0302086](https://arxiv.org/abs/hep-th/0302086)].
- [10] P. Kovtun, D. T. Son and A. O. Starinets, “Holography and hydrodynamics: Diffusion on stretched horizons,” *JHEP* **0310**, 064 (2003) [[arXiv:hep-th/0309213](https://arxiv.org/abs/hep-th/0309213)].
- [11] A. Buchel and J. T. Liu, “Universality of the shear viscosity in supergravity,” *Phys. Rev. Lett.* **93**, 090602 (2004) [[arXiv:hep-th/0311175](https://arxiv.org/abs/hep-th/0311175)].

- [12] P. Kovtun, D. T. Son and A. O. Starinets, “Viscosity in strongly interacting quantum field theories from black hole physics,” *Phys. Rev. Lett.* **94**, 111601 (2005) [arXiv:hep-th/0405231].
- [13] A. Buchel, “ $N = 2^*$ hydrodynamics,” *Nucl. Phys. B* **708**, 451 (2005) [arXiv:hep-th/0406200].
- [14] A. Buchel, J. T. Liu and A. O. Starinets, “Coupling constant dependence of the shear viscosity in $N = 4$ supersymmetric Yang-Mills theory,” *Nucl. Phys. B* **707**, 56 (2005) [arXiv:hep-th/0406264].
- [15] A. Buchel, “On universality of stress-energy tensor correlation functions in supergravity,” *Phys. Lett. B* **609**, 392 (2005) [arXiv:hep-th/0408095].
- [16] P. K. Kovtun and A. O. Starinets, “Quasinormal modes and holography,” arXiv:hep-th/0506184.
- [17] P. Benincasa, A. Buchel and A. O. Starinets, “Sound waves in strongly coupled non-conformal gauge theory plasma,” arXiv:hep-th/0507026.
- [18] O. Aharony, A. Buchel and A. Yarom, “Holographic renormalization of cascading gauge theories,” *Phys. Rev. D* **72**, 066003 (2005) [arXiv:hep-th/0506002].
- [19] A. Buchel, “Transport properties of cascading gauge theories,” arXiv:hep-th/0509083.
- [20] S. S. Gubser, I. R. Klebanov and A. W. Peet, “Entropy and Temperature of Black 3-Branes,” *Phys. Rev. D* **54**, 3915 (1996) [arXiv:hep-th/9602135].
- [21] A. Buchel, “A holographic perspective on Gubser-Mitra conjecture,” arXiv:hep-th/0507275.
- [22] M. T. Grisaru and D. Zanon, “Sigma Model Superstring Corrections To The Einstein-Hilbert Action,” *Phys. Lett. B* **177**, 347 (1986).
- [23] M. D. Freeman, C. N. Pope, M. F. Sohnius and K. S. Stelle, “Higher Order Sigma Model Counterterms And The Effective Action For Superstrings,” *Phys. Lett. B* **178**, 199 (1986).

- [24] Q. H. Park and D. Zanon, “More On Sigma Model Beta Functions And Low-Energy Effective Actions,” *Phys. Rev. D* **35**, 4038 (1987).
- [25] D. J. Gross and E. Witten, “Superstring Modifications Of Einstein’s Equations,” *Nucl. Phys. B* **277**, 1 (1986).
- [26] S. S. Gubser, I. R. Klebanov and A. A. Tseytlin, “Coupling constant dependence in the thermodynamics of $N = 4$ supersymmetric Yang-Mills theory,” *Nucl. Phys. B* **534**, 202 (1998) [arXiv:hep-th/9805156].
- [27] J. Pawelczyk and S. Theisen, “AdS(5) \times S(5) black hole metric at $O(\alpha'^{**3})$,” *JHEP* **9809**, 010 (1998) [arXiv:hep-th/9808126].
- [28] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from non-critical string theory,” *Phys. Lett. B* **428**, 105 (1998) [arXiv:hep-th/9802109].
- [29] E. Witten, “Anti-de Sitter space and holography,” *Adv. Theor. Math. Phys.* **2**, 253 (1998) [arXiv:hep-th/9802150].
- [30] O. DeWolfe and S. B. Giddings, “Scales and hierarchies in warped compactifications and brane worlds,” *Phys. Rev. D* **67**, 066008 (2003) [arXiv:hep-th/0208123].